Drag Reduction in the Turbulent Flow of Fiber Suspensions

An analysis of the velocity profile and pressure drop relationships for turbulent flow of fiber suspensions through smooth tubes was evaluated experimentally over a range of flow rates, tube sizes, fiber concentrations, and fiber geometries (aspect ratios). This work shows that drag reduction in these systems, in marked contrast to that in viscoelastic polymeric fluids, involves processes in the turbulent core of the velocity field. As a result the drag reduction achieved is independent of the scale of the system.

The implications of these results with respect to rates of heat and mass transport are considered in a preliminary way. The measurement of such transport rates, and of the turbulent velocity profiles in dilute suspensions, is seen to be of mechanistic interest.

R. C. VASELESKI
and A. B. METZNER

University of Delaware
Newark, Delaware 19711

SCOPE

Suspensions of solid particles in a liquid or a gas have long been known to decrease the shearing stresses developed as the fluid moves past a solid surface under conditions of turbulent flow. The mechanism of this drag reduction at high Reynolds numbers is not understood fully and work in this area has been largely eclipsed in recent years by studies of dilute polymeric solutions in which the magnitude of the drag reduction effect, for a given additive concentration, may be greater. However, in such viscoelastic polymeric solutions the drag reduction process is essentially a wall effect, and, as a result, it diminishes as the scale of the system is increased at a given Reynolds number. For this reason, as well as for considerations of cost and polymer degradation rates, the many investigations of this phenomenon in laboratory-scale equipment have not resulted in very many practical applications.

Hoyt (1972b) has shown that dilute suspensions of some fibrous solid particles may also exhibit very appreciable drag reduction effects. The purpose of this work was to contribute to a mechanistic understanding of drag reduction in such suspensions.

CONCLUSIONS AND SIGNIFICANCE

Effects within the turbulent core are shown to dominate the drag reduction process in these fiber suspensions. As a result, no effect of tube diameter is observed in the experimental data, using large pipes, over the four-fold range studied. This is in marked contrast to the behavior of polymer solutions.

The velocity profile in the turbulent core may be inferred from the analysis to sharpen progressively as the drag reduction increases. The turbulent profile always remains flatter than the parabola characteristic of laminar flow, of course, but the profile slope may become several times as great as that of a turbulent Newtonian fluid. This is again in marked contrast to the behavior observed in viscoelastic systems, in which no major changes occur within the turbulent core.

Data obtained using macroscopic nylon fibers substantiated the results of previous studies and extended them to include a range of tube diameters. Drag reduction increased as the aspect ratio of the fibers, or their concentration, was increased. However, large concentrations (10,000 ppm) were needed to obtain relatively little drag reduction (8%) with fibers having an aspect ratio of 100.

Measurements using microscopic fibers agreed well with data of previous investigators using small-diameter capillary tubes and extend such measurements to systems having dimensions of commercial interest. Drag reduction is again shown to increase as aspect ratio or fiber concentration increases, in the range studied. Much lower concentrations of asbestos fibers having an aspect ratio of approximately 10^4 are required to produce results comparable to those obtained with the relatively shorter nylon fibers.

HISTORICAL DEVELOPMENT, STATE-OF-THE-ART

Early observations of drag reduction centered on suspensions of natural products such as sand and sediments as well as wood fibers (Blatch, 1906; Vanoni, 1946; Brecht and Heller, 1950; Robertson and Mason, 1957; Maude and Whitmore, 1958; Robertson and Chang, 1967; Zandi, 1967; Manteuffel et al., 1969). Although they were motivated by the need to provide accurate hydraulic transport criteria, the attempts to establish the systematic effects of solids concentration, specific gravity, and of duct diameter were reported as unsuccessful, quite possibly because the suspended particles were not of uniform and reproducible dimensions and surface texture. Progressively, later studies employed well defined synthetic fibers of rayon or nylon to circumvent this difficulty (Daily and Bugliarello, 1958; Schieck, 1958; Daily and Tsuchiya,

Correspondence concerning this paper should be addressed to A. B. Metzner. R. C. Vaseleski is now with Union Carbide, South Charleston, West Virginia.

1959; Bobkowicz and Gauvin, 1965; Mih and Parker, 1967; Kerekes and Douglas, 1972). In dilute systems these studies show that drag reduction increases as the aspect ratio (length: diameter ratio) of the fibers is increased at constant fiber concentration and as concentration is increased at constant fiber L/D. Recent studies by Hoyt (1972b) and Peyser (1972) have accordingly employed fibers with very great aspect ratios and have also reviewed earlier studies of this kind.

The studies of Daily and his co-workers, of Mih and Parker, and of Kerekes and Douglas merit special attention because velocity profile information, important for elucidating mechanistic questions, was sought. Qualitatively, these three studies may be summarized as follows. At low flow rates, the central or core region of the velocity field in a tube consists of a solid plug of flocculated fibers. This is surrounded by an annulus of the suspending liquid (water) which is nearly fiber free. In this regime, the friction factor (or equivalently, the pressure drop) of the suspension is usually greater than that of water alone at the same flow rate because the flatter velocity profile in the core leads to a steeper profile near the boundary, which implies higher wall shearing stresses and thus higher friction factors than those of water. However, as the flow rate increases, the suspension friction factor value drops to levels near, or even below, those of water. Qualitative observations show that this occurs when turbulence sets in at the edge of the annulus of water and tears fibers away from the flocculated plug or core. The velocity profile in the core becomes less blunt progressively as the velocity is increased and the flocculated core is torn apart; the friction factor of the suspension may then drop below the value for water alone at the same flow rate.

These results are interpreted as showing (Robertson and Mason, 1957; Daily and Bugliarello, 1961; Mih and Parker, 1967; Robertson and Chang, 1967) that the presence of the fiber networks in the water suppresses the development and propagation of turbulence and thus reduces the level of momentum transport. At a given velocity, an increase in concentration will yield a greater suppression of turbulence as long as the fibers do not flocculate sufficiently to develop a region of plug flow, with its attendant increase in drag. An increase in fiber L/D should likewise lead to an increasing suppression of turbulence. Presumably the suppression of turbulence by the fibers is related to the extraordinary resistance which the fluid encounters in flow over fibers whenever extensional deformations are important (Batchelor, 1970 and 1971; Mewis and Metzner, 1973) as is the case in the propagation of turbulent velocity fluctuations.

Although this point does not appear to have been emphasized or developed quantitatively in the literature, it is evident from the above discussion that in drag reduction processes involving suspensions of fibers the presence of these fibers in the turbulent core region of the flow is of importance. This is in sharp contrast to the proven mechanisms of drag reduction in polymeric solutions in which effects within the wall region are known to dominate (Wells and Spangler, 1967; Seyer and Metzner, 1969; Hoyt, 1972a). The implications of these different mechanistic origins of drag reduction in suspensions and in polymeric solutions will be developed later in this paper; it is first necessary to verify the qualitative importance of the core-region in a more mechanistic manner and to relate it quantitatively to drag reduction through its influence on pressure drop-flow rate relationships. Further, effects of tube diameter do not appear to have been investigated within any laboratory. It is the purpose of this work to contribute to a resolution of these deficiencies.

THEORETICAL FRAMEWORK

We shall extend, to suspensions, the analytic approach pioneered by Millikan (1939) and later extended to other systems by Dodge and Metzner (1959) and Seyer and Metzner (1969).

The cross section of the tube is assumed to be composed of only two distinct regions under conditions of turbulent flow: a wall region and a turbulent core. Various assumptions can now be made concerning the dependency of mechanism on concentration (c) and aspect ratio ($L/D = \alpha$) of the fibers.

In the wall region the local velocity is assumed to be dependent upon the following variables:

$$u = f_2(\tau_w, y, \rho, \mu, \alpha, c) \quad (1)$$

and in the turbulent core:

$$U_m - u = f_3(\tau_w, y, R, \rho, \alpha, c) \quad (2)$$

Thus over the entire cross section:

$$u = F(\tau_w, y, R, \rho, \mu, \alpha, c) \quad (3)$$

These three equations reduce to those for Newtonian single-phase flows when α and c are set equal to zero. The α and c terms were included in both the wall and core regions for generality of the development.

Introducing the dimensionless groups

$$Z = \frac{Ru^*}{\nu} \quad (4)$$

$$\xi = y/R \quad (5)$$

and using the Buckingham Pi theorem, one obtains the following forms of Equations (1) to (3):

$$\text{Wall region:} \quad u/u^* = f_2(Z\xi, \alpha, c) \quad (6)$$

$$\text{Core region:} \quad \frac{U_m - u}{u^*} = f_3(\xi, \alpha, c) \quad (7)$$

$$\text{Centerline:} \quad U_m/u^* = F(Z, 1, \alpha, c) \quad (8)$$

The equations for the wall region and the core are assumed to coincide at some radial position, so that here

$$u/u^* = f_2 = F - f_3 \quad (9)$$

Equation (9) may be differentiated with respect to radial position in the tube ξ or the dimensionless flow rate Z. The requirement that it remain valid upon such differentiation is equivalent to saying that the core and wall region profiles must join smoothly and must remain joined as the flow rate is changed. Such manipulation enables one to fix the general form of the profile equations; Equation (9) may thus be shown to lead to

$$f_2(Z\xi, \alpha, c) = u/u^* = A(\alpha, c) \ln Z\xi + B(\alpha, c) - C'(\alpha, c) \quad (10)$$

Equation (10) is valid only in the region of overlap due to the assumption made in deriving it but is commonly employed over the entire core region by allowing $C'(\alpha, c)$ to depend also upon ξ. Following the arguments of Seyer and Metzner (1969) the value of $C'(\alpha, c, \xi)$ can be taken as zero as a first approximation since it is identically zero at the centerline and makes a negligible contribution to the velocity term in Equation (10) in the range $0.3 < \xi < 1.0$. For values of ξ less than 0.3, the C' function generally makes less than a 10% contribution to the velocity. Introducing this approximation the velocity profile reduces to

$$u/u^* = A(\alpha, c) \ln Z\xi + B(\alpha, c) \quad (11)$$

or

$$u^+ = A \ln y^+ + B$$

By definition, the Fanning friction factor is given as

$$f = \frac{\tau_w g_c}{\rho V^2/2} = 2\left(\frac{u^*}{V}\right)^2 \quad (12)$$

in which the average velocity V is defined as

$$V = \left(\int_0^R 2r\, u(r)\, dr \right) \Big/ R^2 \quad (13)$$

In systems in which the sublayer thickens upon introduction of a drag reducing additive it is necessary to integrate Equation (13) in a piece-wise manner. In the core region Equation (11) will be used. The edge of the sublayer is denoted by y_1^+ and it is assumed that $u^+ = y^+$ in the sublayer defined as $0 < y^+ < y_1^+$. Splitting the integral of Equation (13) at ξ_1, (the point of intersection of the sublayer and core regions of the velocity profile) integrating and ignoring terms in ξ_1^3 yields

$$[U_m(1-\xi_1)^2 - V]/u^* = G\left(\xi_1, \alpha, c, \frac{Ru^*}{\nu}\right) \quad (14)$$

Substituting Equations (11) and (12) into Equation (14) yields

$$\sqrt{2/f} = (1-\xi_1)^2 A(\alpha, c) \ln (Re\sqrt{f}) + (1-\xi_1)^2 [B(\alpha, c) - A(\alpha, c) \ln 2\sqrt{2}] - G\left(\xi_1, \alpha, c\frac{Ru^*}{\nu}\right) \quad (15)$$

Equations (11) and (15) are the principal results of the present analysis. They are important in interrelating the turbulent velocity profile and the pressure drop characteristics of the flow process through the parameters A and B. This assumes the terms ξ_1 and G can be evaluated; constancy of the slope of a plot of $1/\sqrt{f}$ versus $Re\sqrt{f}$ for a given fluid would certify that ξ_1 is negligibly small and that G does not depend measurably upon Ru^*/ν. It will be seen that this is indeed the case. Seyer and Metzner assumed a constant value of 3.00 for G, since it has a value of 3.60 in Newtonian fluids and decreases as ξ_1, the sublayer thickness, increases [Equation (14)], an effect which may be expected in drag reducing systems involving polymeric additives. In the limit of vanishing c and α Equation (15) must reduce to the Newtonian equation. Thus $A = 2.46$ and $B = 4.96$ (instead of $B = 5.6$, since G was chosen to be 3.00 instead of the true Newtonian value of 3.60). Plots of $1/\sqrt{f}$ versus log $Re\sqrt{f}$, as a function of concentration and aspect ratio of the fibers, will allow the dependency of A and B on c and α to be determined.

Mechanistically, one wishes to define the origin of the drag reduction effects as precisely as possible. To do so, the above analysis was repeated 8 additional times, each time varying the basic assumptions concerning the role of the parameters α and c in Equations (1) to (3). These assumptions, and the results of each analysis, are all summarized in Table 1. One sees that an experimental evaluation of the dependency of A upon α and c will enable distinction between the first 4 assumptions, the 5th, the 6th and 9th, or the 7th and 8th. This is not as complete or sharp a discrimination as desirable but does enable a focusing of further research and will prove to be of some value in its own right. Unfortunately the experimental evaluation of B adds nothing of value to increase our discrimination. It is interesting to note in passing that in all cases except #5 the slope of the velocity profile in the turbulent core (A) is predicted to change upon the addition of fibers, in contrast to the behavior of polymeric additives, for which the turbulent core remains unaffected.

TABLE 1. DEPENDENCY OF SLOPES AND INTERCEPTS [EQUATION 15] UPON ASSUMPTIONS USED IN DERIVATION

	If wall region is a function of	and core region is a function of	Then A and G are functions of	and B is a function of . . . .
1.	α, c	α, c	α, c	α, c
2.	—	α, c	α, c	α, c
3.	α	α, c	α, c	α, c
4.	c	α, c	α, c	α, c
5.	α, c	—	—	α, c
6.	α, c	α	α	α, c
7.	α, c	c	c	α, c
8.	α	c	c	α, c
9.	c	α	α	α, c

EXPERIMENT

A series of friction loss experiments utilizing pressure drop and flow rate readings were performed using fiber suspensions which were pumped continuously through a test loop in turbulent flow. The test fluid was pumped through one of four test sections, each 6.1 m long and consisting of 1 in., 2 in., 3 in., or 4 in. smooth polyvinyl chloride (PVC) pipes. The inside diameters of these pipes were 2.42, 4.87, 7.03, and 9.50 cm and the respective entrance lengths were 82, 41, 28, and 21 diameters. The distance between pressure taps was 3 m. It is shown in the dissertation (Vaseleski, 1973) that the results are free of entrance-length and pressure tap errors.

Flow rates were measured using a calibrated Foxboro Magnetic Flowmeter and pressure drops were obtained manometrically. The manometer leads were of clear plastic to allow observance of trapped air bubbles or fibers and were equipped with air bleed lines and a tap water purge to prevent clogging.

In order to elucidate the effects of fiber aspect ratio α nylon fibers were chosen with aspect ratios varying between 73 and 141 (previous studies covered the range 17 to 74 in addition to one set of data at $\alpha = 370$). To obtain much greater aspect ratios, of interest since these should be much more effective drag reducing additives if the earlier qualitative mechanistic comments are correct, chrysotile asbestos fibers were also used. It has been reported (Atkinson et al., 1970) that these may possess aspect ratios as great as 10^5. The expectation of the superior drag reduction properties of these fibers have been borne out by several studies (Arranaga, 1970; Ellis, 1970; Hoyt, 1972b; Peyser, 1972) using capillary tubes and rotating disks. Our asbestos samples were obtained from Johns-Manville, Asbestos, Quebec, Canada, as a dry fiber labeled 3T12 and from Turner Brothers, Rochdale, England, as a 1.5% dispersion in a 0.8% surfactant solution. The surfactant used to disperse both asbestos types was obtained from Fisher Scientific Company, Philadelphia, Pennsylvania, as Aerosol OT Powder. A 0.25% surfactant solution was used for the JM asbestos and a 0.8% solution for that from Turner Brothers.

Slurry preparation involved draining, flushing, and refilling the storage tank and test loop. The surfactant was then added, mixed, and tested as to its Newtonian drag coefficient—Reynolds number behavior. The fibers were then added and dispersed by mixing and overnight circulation through the test loop.

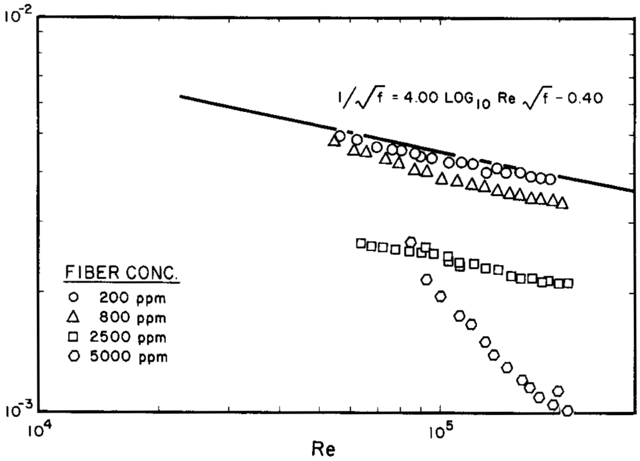

Fig. 1. Data for JM Asbestos fibers in 4.9-cm pipe.

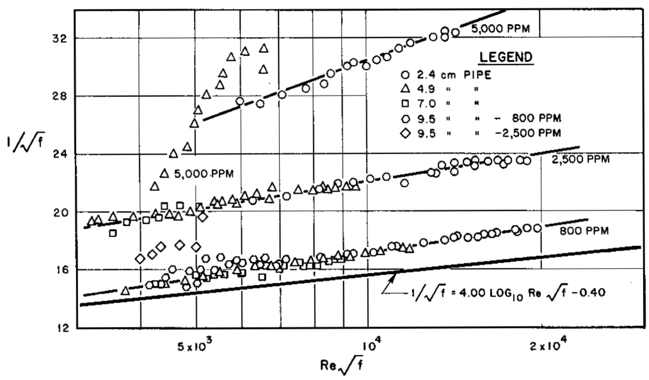

Fig. 2. Data for JM Asbestos fibers: all pipe diameters.

RESULTS AND DISCUSSION

The measured tube diameters were checked and adjusted utilizing turbulent flow results together with the von Karman equation:

$$1/\sqrt{f} = 4.0 \log_{10}(Re\sqrt{f}) - 0.40 \qquad (16)$$

All subsequent data for water and for both the 0.25 and 0.80% surfactant solutions were shown to agree closely with Equation (16), which was, therefore, always used as the reference line for purposes of determining the percentage reduction in drag coefficient.

Data for nylon suspensions obtained by Bobkowicz and Gauvin (1965) and Kerekes and Douglas (1972), as well as those of the present study, were analyzed using the theoretical framework presented earlier. While it could be shown that drag reduction increased with increasing fiber concentration and aspect ratio, the all-important slopes and intercepts of the $1/\sqrt{f}$ versus $(Re\sqrt{f})$ plots necessary to verify Equation (15) and to differentiate between the various alternatives of Table 1 simply could not be ascertained except in one case. At low concentrations the actual drag reduction and the deviations from Equation (16) were too small to be determined precisely. At higher concentrations the range of Reynolds number which could be studied became too small because of delays in transition to fully developed turbulence.

Figure 1 shows typical results for JM asbestos fibers; at 5000 ppm the transition to turbulence is evidently delayed to Reynolds numbers beyond 2×10^5; such delays in transition were generally observed and became even greater in the larger tubes. In all cases the Reynolds number shown is based on the measured continuum viscosity of the surfactant solution; complete rheological data for the suspensions in shearing flows are given by Lee, Vaseleski, and Metzner (1974).

Figure 2 depicts the data for these suspensions in the format suggested by Equation (15); data for the 100 ppm and 200 ppm concentration levels are omitted for reasons of clarity. All data show definite increases in drag reduction with increases in fiber concentration level and, within experimental accuracy, there is no effect of tube diameter. This latter point rules out any significant slip or other wall region effects. That is, as far as can be ascertained from pressure drop measurements, Case 2 of Table 1 tends to be suggested. However, more subtle effects within the wall region, which do exert any measurable effect on pressure drop, cannot be ruled out at this point. The transitional nature of the 2500 ppm data in the 9.5-cm line and of the 5000 ppm data in the 4.9-cm line are revealed clearly. These suspensions showed no degradation with time as the measurements shown in Figure 2 are a composite of measurements over a period of days of pumping, with many repeated measurements. Finally, the constancy of the slopes shows that the bothersome ξ_1 terms in Equation (15) must be negligibly small and that the parameter G is not a function of Reynolds number.

In addition to the results shown in Figure 2, measurements were made on a second asbestos system and on nylon suspensions, over a range of pipe sizes, and 2 sets of rayon suspension data are available from the literature. The results of all these studies are summarized in Table 2. As is evident from both the rayon and asbestos data, the parameter A increases with increasing fiber concentration. This immediately rules out cases 5, 6, and 9 (Table 1): concentration must have an effect on the turbulent core, and its effect is to steepen the logarithmic velocity profiles—Equation (11). Cases 7 and 8 of Table 1 appear to be ruled out by comparison of the rayon and JM data also: the parameter A, at a given concentration level, is evidently dependent on the fiber aspect ratio. [The aspect ratio of the asbestos is not known precisely, but one may infer from Hoyt (1972b) that it is at least an order of magnitude greater than that of the rayon fibers]. However, presumably the TB asbestos changed in aspect ratio upon degrading, yet no effect is found upon A. Thus although Cases 7 and 8 appear unlikely, they are not ruled out conclusively. Comparing the several systems studied, Turner Brothers asbestos gave better drag reduction effects than did the JM fibers (15% at 100 ppm as compared to 17% at 800 ppm), which in turn were superior to nylon suspensions (8% at 10,000 ppm). As in the study of Hoyt (1972b) only the Turner Brothers asbestos degraded with time. In no case did the drag reduction depend upon tube diameter over the 4-fold range studied.

It is instructive to examine the velocity profiles, Equation (11), more closely now that the parameter A is known precisely and B at least approximately. Equation (11), together with the usual laminar $u^+ = y^+$ sublayer curve, is plotted in Figure 3. One must proceed with caution, as the curves are predicted, not measured. The velocity profile in the turbulent core is predicted to be dramatically steeper in suspensions than in Newtonian fluids. This is because of the lowered rates of turbulent momentum transport here. More interesting is the point of apparent intersection of the sublayer and turbulent core profiles. At least in the 0 to 800 ppm concentration range this point appears to move progressively closer to the tube wall, as suspension concentration increases, implying a thinner sublayer with a larger value of G [Equation (15)] and with improved heat and mass transport

TABLE 2. SUMMARY OF RESULTS

Additive fibers	Conc., ppm	Fiber aspect ratio, α	Approx. percentage drag reduction	Intercept of Equation (15), $B-G-A\ln 2\sqrt{2}$	A	B**
None	0	—	0	−0.40	2.45	5.0
Rayon*	2,000	370	13	−0.54	2.62	5.2
	5,000	370	18	−0.52	2.70	5.3
Nylon	10,000	100	8	−2.85	2.82	3.1
JM Asbestos	100	?	2	−3.67	2.80	2.2
	200	?	7	−5.74	3.10	0.5
	800	?	17	−7.35	3.45	−0.8
	2,500	?	46	−6.87	4.12	0.4
	5,000	?	70	−40.8	(9.07)	(−28.4)
Turner Brothers asbestos	100	4×10^4	15	2.4	2.85	3.6
	200	4×10^4	27	1.2	3.00	4.9
	200†	?	15	3.5	3.02	2.6

* Data of Mih and Parker (1967).
† Degraded by pumping.
** Evaluated assuming $G = 3.0$.

rates. This is a most tentative conjecture, for in actuality the two-region approximation must be replaced by one permitting a buffer-layer between the wall region and the core. It is, nevertheless, an intriguing conjecture for development of improved process fluids which exhibit reduced power requirements and improved heat or mass transport rates concomitantly.

The 5,000-ppm curves do not show any point of intersection between the sublayer and core equations. A very broad buffer zone connecting the two regions is entirely reasonable, but the lack of any apparent point of intersection violates a key assumption used in the development, making the profile prediction shown more tenuous than for the other systems. For this reason the A and B values in Table 2 are shown in parentheses. Clearly both direct measurements of the profiles, and of heat transfer rates, would be of much interest.

Finally, the dramatic mechanistic differences between drag reduction in suspensions and that in polymeric solutions leads to one further interesting question: Can both mechanisms be exploited together? The answer appears to be at least partially affirmative and is dealt with elsewhere (Lee, Vaseleski, and Metzner, 1974).

ACKNOWLEDGMENT

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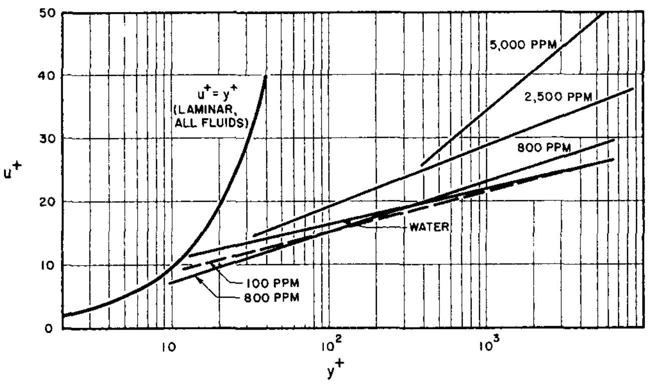

Fig. 3. Predicted turbulent velocity profiles—JM fiber suspensions.

NOTATION

A = slope of logarithmic velocity profile
B, C' = intercept functions, Equations (10) and (11)
c = concentration in ppm by weight
D = tube diameter
f = Fanning friction factor
f_2, f_3, F = functions to be determined experimentally
G = function, Equation (14)
L/D = aspect ratio (length/diameter) of fiber
r = radial position, $R-y$
R = radius of tube
Re = Reynolds number, $DV\rho/\mu$
u^* = friction velocity, $\sqrt{\tau_w/\rho}$
u^+ = dimensionless velocity, u/u^*
u = local time averaged axial velocity
U_m = centerline velocity
V = bulk velocity
y = distance from tube wall
y^+ = dimensionless distance from tube wall, $Z\xi$
y_1^+ = y^+ at edge of sublayer
Z = dimensionless Reynolds number, Ru^*/ν
α = aspect ratio, L/D
μ = viscosity
ν = kinematic viscosity
ξ = dimensionless distance from tube wall, y/R
ξ_1 = ξ evaluated at edge of sublayer
ρ = density
τ_w = shear stress at tube wall

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Separation of Multicomponent Mixtures via Thermal Parametric Pumping

A thermal continuous parametric pump for separating multicomponent mixtures was experimentally investigated using the model system toluene-aniline-*n*-heptane on silica gel adsorbent. A simple method for predicting separations is presented and is found to be in good agreement with the experimental results. The method, based on an equilibrium theory, invokes the assumption that a multicomponent mixture contains a series of pseudo binary systems. Each binary system consists of one of the solutes as one component and the common inert solvent as the other component.

H. T. CHEN
W. W. LIN
J. D. STOKES
and W. R. FABISIAK

Newark College of Engineering
Newark, New Jersey 07102

SCOPE

Thermal parametric pumping is a phase-change separation process which depends for its operation on the coupling of periodic changes in temperature affecting the position of interphase equilibrium with synchronous periodic changes in flow direction. In earlier papers (Chen et al., 1972, 1973), separations of binary systems (single solute and its solvent) were experimentally investigated via continuous and semicontinuous parametric pumping. It has been shown that under certain conditions the pumps with feed at the enriched end have the capacity for complete removal of solute from one product stream and, at the same time, give arbitrarily large enrichment of solute in the other product stream.

In this paper continuous parametric pumping is extended to the separations of multicomponent mixtures. The continuous pump is characterized by a steady flow of both feed and product streams during the hot upflow and cold downflow half-cycles. The system used is toluene-aniline-*n*-heptane on silica gel. A comparison is made between the experimental data and the calculated results based on the method proposed by Chen and Hill (1971). The prediction of multicomponent separations by this relatively simple method could eventually be applied to commercially important systems which are usually multicomponent in nature. It should be noted that a theoretical discussion of separations of multicomponent mixtures by other versions of parametric pumping has been given by Butts et al. (1972).